

# Allocating advertising expenses using Linear Programming and MS –Excel

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## Summary

One of the most important functions of enterprises is advertising as a means of systematically approaching specific market segments and informing prospective customers for their products, services or even their existence. The purpose of every advertising campaign is to achieve the greatest possible impact, to the market segments addressed, within the frames of a given budget and under a set of qualitative or quantitative constraints. The cost to benefit relationship of such a campaign is an important issue for every enterprise and in particular for small or medium ones. Linear programming techniques can contribute towards the effective allocation of advertising expenses and the use of MS Excel Solver facilitates the resolution of the resulting mathematical models.

**Keywords:** Advertising expenses, Linear Programming, MS Excel Solver

## 1 The use of Linear Programming in allocating advertising expenses

The issue of developing and using linear models for the optimal allocation of advertising expenses among available advertising vehicles has been extensively debated [Zangwill (1965)], [Brown and Warshaw (1965)], [Bass and Lonsdale (1966)], [Stasch (1965) and (1967)], [Keown and Duncan (1974)], [Caine and Robson (1993)]. On the basis of the relevant literature and given that this can be easily formulated as a typical resource allocation problem, one would expect that Linear Programming (LP) techniques would have been widely used in this respect. However, although these techniques are taught in every business or management school worldwide, they have been, so far, hardly used in real world conditions by management. This is because the LP formulation of, even quite simple, business situations involves an exceptionally big number of variables and constraints and, hence, expensive dedicated software requiring specialised personnel needed to be used for handling the resulting models. Thus LP, for a number of years, has been used only by very big business, government agencies and organizations or in the frames of academic research [Caine and Parker (1996)].

The extensive use of personal computers, the dramatic reduction of their cost and the tremendous increase of their computing ability have influenced the management culture worldwide. Senior, medium and front-line management have now access to personal computers and spreadsheet software such as Microsoft Excel [Microsoft Corporation, (1985-2001)] is extensively used. The package contains Solver, an exceptionally evolved and impressively powerful tool that is very effective for handling linear and non linear optimisation problems [Burton,

Carrol & Wall (1999)]. Resource allocation situations are not only easily handled by Solver but additional decision support information can also be obtained [Caine and Parker (1996)]. In the following pages the optimal distribution of advertising expenses using Linear Programming and Microsoft Excel's Solver is described.

## 2 The Problem

A small commercial enterprise in Heraklion, Crete asked a local agency to design and implement, on its behalf, an advertising campaign for the August – September period of 2003 within the frames of a budget of € 4.000,00. The agency proposed as the most suitable advertising vehicles for the occasion the 4 local daily newspapers and its creative department produced a two column, 10 cm long storyboard that was accepted by the company.

The agency's people asked our assistance regarding the distribution of advertising expenses among the newspapers (which for obvious reasons we will, from now on, call Newspaper\_1, 2, 3 and 4) in order to achieve optimal effectiveness within the frames of the available budget. For keeping relations with newspapers (although Heraklion is a capital city its business community is a quite close caste) the company required that Newspaper\_2 should get at least 15% of the total no of ads to be published, while Newspaper\_1 and 4 should get at least 10% each.

## 3 Available Data

Newspapers\_2 and 3 are independent and are mainly distributed through the network of news kiosks and stands while Newspapers\_1 and 4 are closely connected to specific political parties and are distributed mainly to subscribers which are principally enterprises, professional offices and public services and organisations. In the following Table 1, the circulations of the 4 newspapers according to the local Distribution Agency are presented

Table 1. Circulations

Newspaper	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
<b>Np_1</b>		2.000	2.000	2.000	2.000	2.000	2.000
<b>Np_2</b>	10.000	6.000	6.000	6.000	6.000	6.000	
<b>Np_3</b>		12.000	7.000	7.000	7.000	7.000	7.000
<b>Np_4</b>		1.700	1.700	1.700	1.700	1.700	1.700

Marketing people know that each newspaper copy is usually read by more, than one, readers. According to data available to the advertising agency, the average readers per copy are 4 for Np\_1, 2 and 4 and 6 for Np\_3. So, to calculate total newspaper readerships, circulations should be multiplied by the relevant readers/copy factors.

Newspaper advertising space unit is the "running centimetre" i.e. a vertical centimetre of a single column. Thus a 10cm long, two columns wide ad will be charged as 20 running centimetres e.t.c. It is considered that a front page ad will be read by 100% of the newspaper's readership (Exposure factor = 1,00), the same ad in the back page will attract 80% of the same readership (Exposure factor = 0,80) while in internal pages the percentage of readers attracted by an ad is limited to 40% (Exposure factor = 0,40). Based on this, the price lists of the 4 newspapers are listed in the following Table 2.

Table 2. Advertising Cost (running centimetre)

Newspaper	Front Page	Back Page	Internal Pages
<b>Np_1</b>	€ 6,00	€ 4,00	€ 3,00
<b>Np_2</b>	€ 8,00	€ 6,00	€ 3,00
<b>Np_3</b>	€ 8,00	€ 6,00	€ 3,00
<b>Np_4</b>	€ 6,00	€ 4,00	€ 3,00

## 4 Methodology and Solution

To measure the effectiveness of each available advertising vehicle we have used Weighted Adjusted Audience (WAA) [Bass and Lonsdale (1966)] that results from the following formula.

$$\text{WAA} = \text{Adjusted audience (AA)} \times \text{Exposure factor (ExF)} \times \text{Evaluation Factor (EvF)}$$

Where AA is the total readership or audience of the advertising vehicle, less that part of it not thought to be customers of the product or service, ExF is the fraction of the readership or audience one expects to see the advertisement and EvF is a subjective weight, scored on a 0-1 scale by an advertising expert, representing the appropriateness of the advertising vehicle in question. For each one of the four newspapers we have considered three separate advertising vehicles i.e. front page, back page and internal pages on the grounds of different Exposure Factors. Because of their increased circulation the Monday's edition of Np\_2 and Tuesday's edition of Np\_3 were treated as separate newspapers. Thus a total of 18 advertising vehicles were available. The following Table 3 summarizes all relevant information.

Table 3. Advertising vehicles information

Advertising Vehicle	Cost (€) per 10cm Two column Ad	Circulation	Readers/ copy	AA*	ExF	EvF**	WAA
Np_1 FP	120	2.000	4	8.000	1,00	0,80	6.400
Np_1 IP	60	2.000	4	8.000	0,40	0,80	2.560
Np_1 BP	80	2.000	4	8.000	0,80	0,80	5.120
Np_2 FP	160	6.000	4	24.000	1,00	1,00	24.000
Np_2 IP	60	6.000	4	24.000	0,40	1,00	9.600
Np_2 BP	120	6.000	4	24.000	0,80	1,00	19.200
Np_2 (Mon) FP	160	10.000	4	40.000	1,00	1,00	40.000
Np_2 (Mon) IP	60	10.000	4	40.000	0,40	1,00	16.000
Np_2 (Mon) BP	120	10.000	4	40.000	0,80	1,00	32.000
Np_3 FP	160	7.000	6	42.000	1,00	1,00	42.000
Np_3 IP	60	7.000	6	42.000	0,40	1,00	16.800
Np_3 BP	120	7.000	6	42.000	0,80	1,00	33.600
Np_3 (Tue) FP	160	12.000	6	72.000	1,00	1,00	72.000
Np_3 (Tue) IP	60	12.000	6	72.000	0,40	1,00	28.800
Np_3 (Tue) BP	120	12.000	6	72.000	0,80	1,00	57.600
Np_4 FP	120	1.700	4	6.800	1,00	0,70	4.760
Np_4 IP	60	1.700	4	6.800	0,40	0,70	1.904
Np_4 BP	80	1.700	4	6.800	0,80	0,70	3.808

\* No evidence to support exclusion of any part of readership

\*\* Subjective evaluation of advertising vehicles by a marketing expert

### 4.1 Linear Programming Formulation

Variables D2 to D19 represent the number of ads in available advertising vehicles, B2 to B19 are the corresponding costs and C2 to C19 the associated WAA's. The Linear Programming Formulation of the problem will be the following:

#### 4.1.1 Objective function

$$(1) \quad \text{Maximize total WAA} = \sum_{i=2}^{19} C_i D_i$$

### 4.1.2 Constraints

(2) Non-negativity  $D_i \geq 0, \quad i = 2 \text{ to } 19$

(3) Integrity  $D_i = \text{integer}, \quad i = 2 \text{ to } 19$

(4) Budget  $\sum_{i=2}^{19} B_i D_i \leq \text{€}4.000,00$

During the period from 1/8 – 30/9/2003, Np\_1 and 4 will publish 50 issues each, Np\_2 will publish 9 Monday issues and 42 in other weekdays and Np\_3 will publish 9 Tuesday and 41 other issues. Hence operational constraints (5) to (10), preventing an ad to be published more than once in each issue, emerge.

(5) Total Np\_1 ads  $\sum_{i=2}^4 D_i \leq 50$       (6) Total Np\_2 ads  $\sum_{i=5}^7 D_i \leq 42$

(7) Total Np\_2 (Mon) ads  $\sum_{i=8}^{10} D_i \leq 9$       (8) Total Np\_3 ads  $\sum_{i=11}^{13} D_i \leq 41$

(9) Total Np\_3 (Tue) ads  $\sum_{i=14}^{16} D_i \leq 9$       (10) Total Np\_4 ads  $\sum_{i=17}^{19} D_i \leq 50$

(11) Allocation of ads to Newspaper 1  $\sum_{i=2}^4 D_i \geq 10\% \times \sum_{i=2}^{19} D_i$

(12) Allocation of ads to Newspaper 2  $\sum_{i=5}^{10} D_i \geq 15\% \times \sum_{i=2}^{19} D_i$

(13) Allocation of ads to Newspaper 4  $\sum_{i=7}^{19} D_i \geq 10\% \times \sum_{i=2}^{19} D_i$

### 4.2 Using Excel Solver

The transformation of the LP formulation into Excel format is presented in the following Figure 1.

In the same Figure 1 the solution of the problem is also presented. It can be easily observed that by totally spending its available budget of € 4.000,00, the company can achieve a maximum of 1.226.800 WAA, by publishing a total of 30 ads under the given set of constraints.

The optimum advertising scheme is as follows: Publish 3 ads in the internal pages of Np\_1, 9 in the front page of the Monday edition of Np\_2, 6 in the back page of Np\_3, 9 in the front page of the Tuesday edition of the same paper and 1 in the internal and 2 in the back pages of Np\_4. This scheme also satisfies all constraints (see the constraints section of Figure 1).

Figure 1. Transformation into Excel Format

	A	B	C	D
3	<b>Np_1 IP</b>	60	2.560	<b>3</b>
4	<b>Np_1 BP</b>	80	5.120	<b>0</b>
5	<b>Np_2 FP</b>	160	24.000	<b>0</b>
6	<b>Np_2 IP</b>	60	9.600	<b>0</b>
7	<b>Np_2 BP</b>	120	19.200	<b>0</b>
8	<b>Np_2 (Mon) FP</b>	160	40.000	<b>9</b>
9	<b>Np_2 (Mon) IP</b>	60	16.000	<b>0</b>
10	<b>Np_2 (Mon) BP</b>	120	32.000	<b>0</b>
11	<b>Np_3 FP</b>	160	42.000	<b>0</b>
12	<b>Np_3 IP</b>	60	16.800	<b>0</b>
13	<b>Np_3 BP</b>	120	33.600	<b>6</b>
14	<b>Np_3 (Tue) FP</b>	160	72.000	<b>9</b>
15	<b>Np_3 (Tue) IP</b>	60	28.800	<b>0</b>
16	<b>Np_3 (Tue) BP</b>	120	57.600	<b>0</b>
17	<b>Np_4 FP</b>	120	4.760	<b>0</b>
18	<b>Np_4 IP</b>	60	1.904	<b>1</b>
19	<b>Np_4 BP</b>	80	3.808	<b>2</b>
20	<b>Objective function</b>	<b>Maximize total WAA</b>		<b>1.226.800</b>
21	<b>Subject to the following constraints</b>			
22	<b>Non-negativity</b>			<b>0</b>
23	<b>Integer solution</b>			
24	<b>Budget</b>	<b>€ 4.000,00</b>	<b>no more than</b>	<b>€ 4.000,00</b>
25	<b>Operational Constraints</b>			
26	<b>Total No of Ads in Np_1</b>	<b>3</b>	<b>no more than</b>	<b>50</b>
27	<b>Total No of Ads in Np_2</b>	<b>0</b>	<b>no more than</b>	<b>42</b>
28	<b>Total No of Ads in Np_2 (Mon)</b>	<b>9</b>	<b>no more than</b>	<b>9</b>
29	<b>Total No of Ads in Np_3</b>	<b>6</b>	<b>no more than</b>	<b>41</b>
30	<b>Total No of Ads in Np_3 (Tue)</b>	<b>9</b>	<b>no more than</b>	<b>9</b>
31	<b>Total No of Ads in Np_4</b>	<b>3</b>	<b>no more than</b>	<b>50</b>
32	<b>Allocating busines constraints</b>			
33	<b>Total No of Ads in Newspaper 1</b>	<b>3</b>	<b>at least</b>	<b>3</b>
34	<b>Total No of Ads in Newspaper 2</b>	<b>9</b>	<b>at least</b>	<b>5</b>
35	<b>Total No of Ads in Newspaper 4</b>	<b>3</b>	<b>at least</b>	<b>3</b>

## 5 Conclusions

Linear Programming is a widely known decision-making technique, often the only suitable in several resource allocation situations. Its use has been prevented because the LP formulation of most real world business problems requires complex mathematical computations. For many years, available software capable of handling these computations was either too expensive or difficult to be used by non-specialised management staff and hence, the technique was used only by big business, government agencies and academics.

The methodological approach for the distribution of advertising expenses presented in this paper could be extended to include a variety of available advertising vehicles such as radio and television spots, magazine ads e.t.c., provided that information allowing enterprises and advertising agencies to quantify their relative effectiveness exists. Additional data concerning audience overlapping and advertising vehicles' effectiveness in addressing various market segments will produce even more valuable solutions. Local business bodies e.g. the chamber of commerce, advertising agencies or market research companies could initiate and finance research projects in order to refine and renew existing data and produce additional information in this respect.

Given that MS Excel, or other similar packages, provides decision-makers with extremely powerful tools capable of handling complex linear and non linear optimization problems, advertising expense allocation problems formulated in LP format can be easily handled, with no need of either specialised software or personnel, at almost no cost.

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